ECE 344

MICROWAVE FUNDAMENTALS PART1-Lecture 2

Dr. Gehan Sami

Transmission-Line Theory

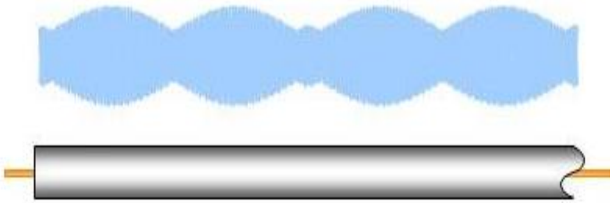
- Lumped circuits: resistors, capacitors, inductors

  electrical wavelength is much larger than the physical dimension of the circuits

  neglect time delays (phase)

- Distributed circuit elements: transmission lines

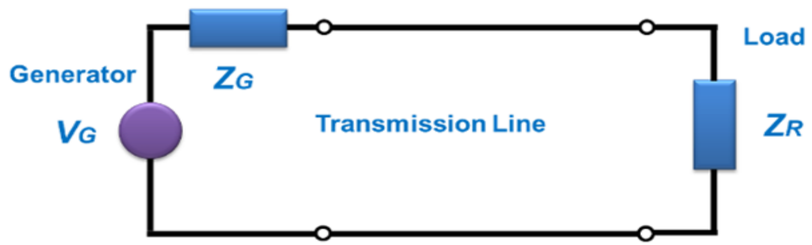
  account for propagation and time delays (phase change)



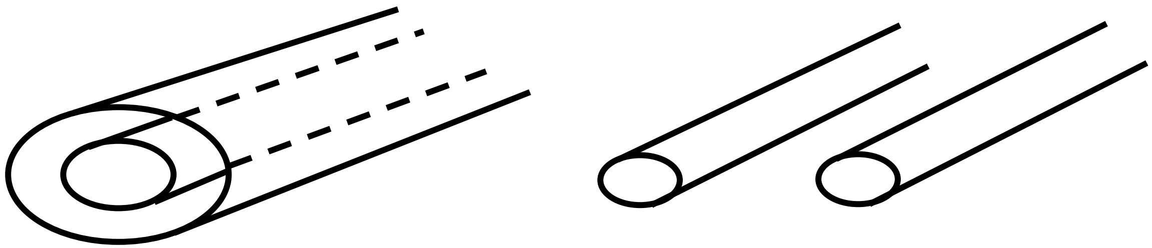
We need transmission-line theory whenever the length of a line is significant compared with a wavelength.

Transmission Line

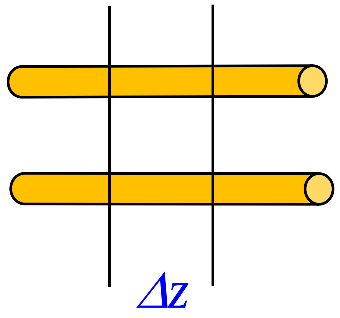
An engineering problem is to transfer signal from generator to load.
A transmission line is a part of circuit that link between generator and load; Theory of transmission line applied on all types of transmission lines.



2 conductors



4 per-unit-length parameters:



C = capacitance/length [F/m]

L = inductance/length [H/m]

R = resistance/length [Ω/m]

G = conductance/length [℧/m or S/m]

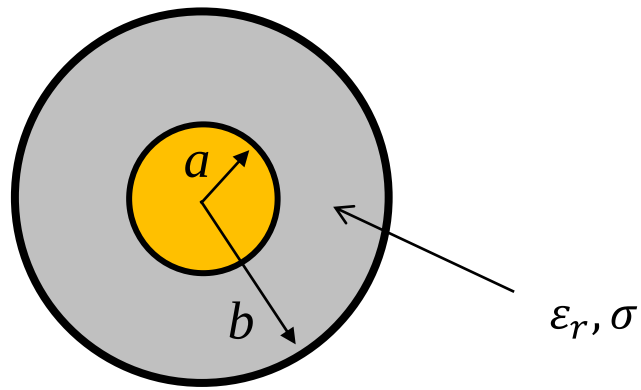
Coaxial Cable

Characteristic impedance depend on dimension and dielectric material

$$C = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln\left(\frac{b}{a}\right)} \quad [\mathrm{F/m}]$$

$$L = \frac{\mu_0\mu_r}{2\pi}\ln\left(\frac{b}{a}\right) [\mathrm{H/m}]$$

$$LC = \mu\varepsilon = \mu_0\varepsilon_0(\mu_r\varepsilon_r)$$



For lossless TL

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{\mu}{\varepsilon}}\ \frac{1}{2\pi}\ln\left(\frac{b}{a}\right) \ [\Omega]$$

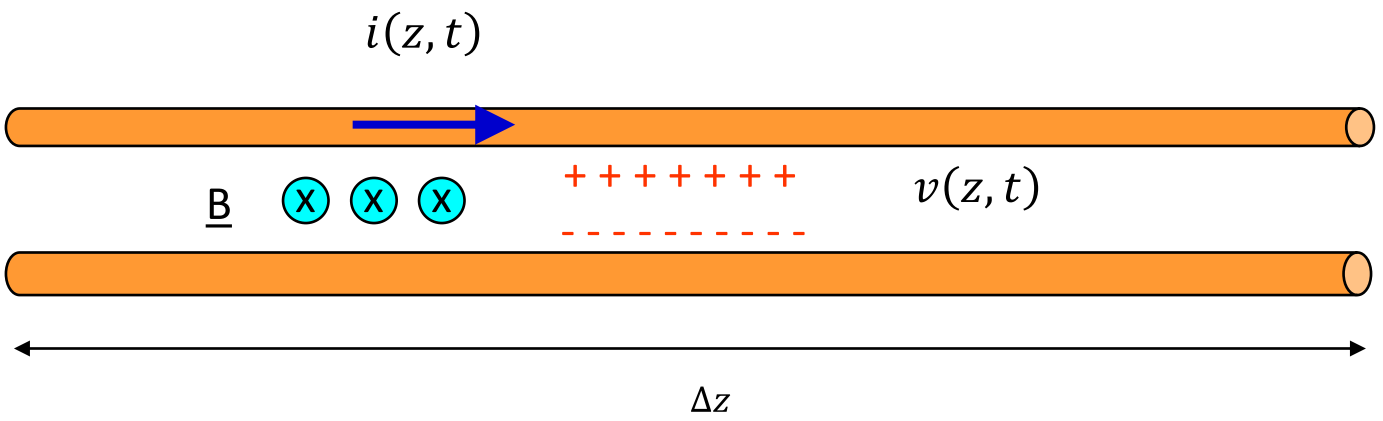
$$G = \frac{2\pi\sigma}{\ln\left(\frac{b}{a}\right)} [\mathrm{S/m}]$$

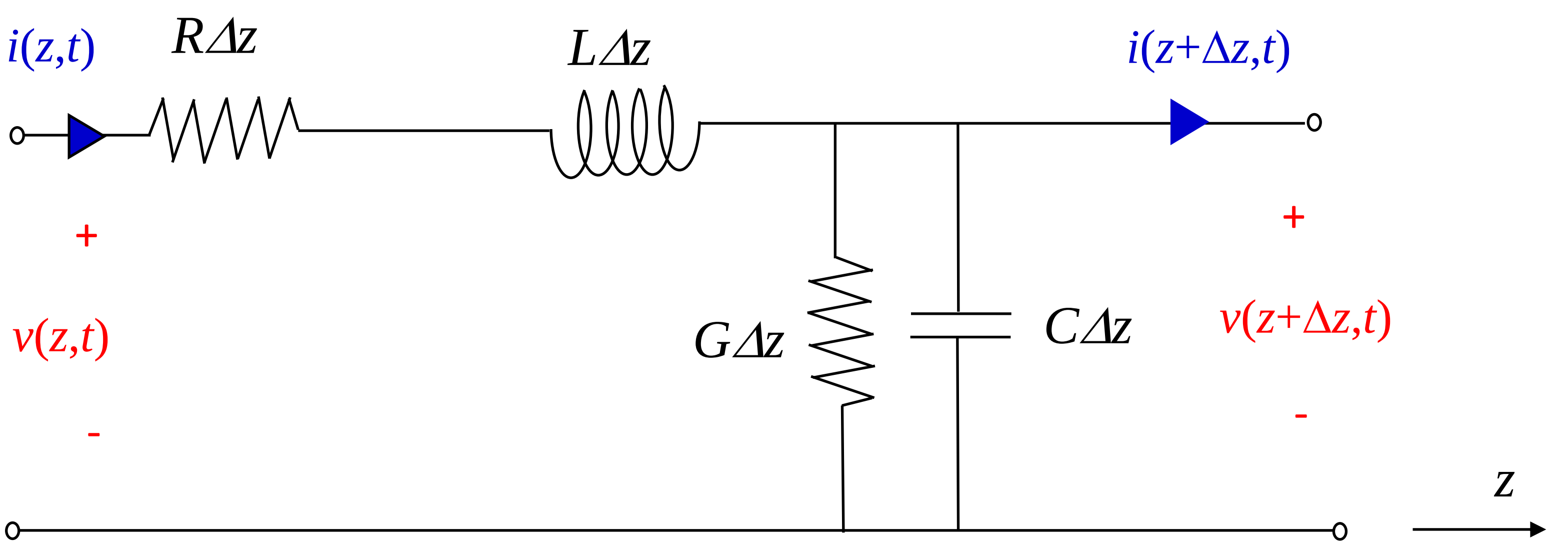
$$R = R_s\left(\frac{1}{2\pi a} + \frac{1}{2\pi b}\right)$$

$$R_s = \frac{1}{\sigma\ \delta}$$

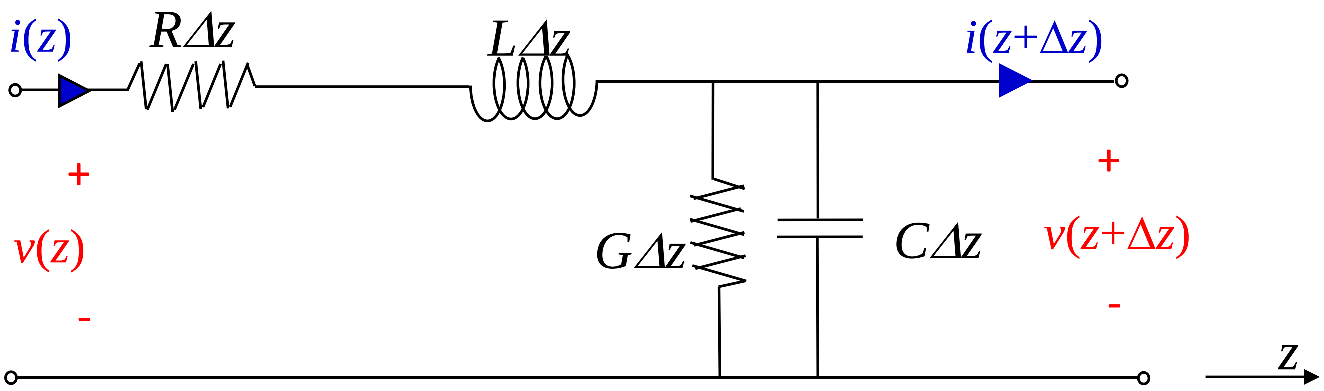
$$\delta = \sqrt{\frac{2}{\omega\mu_0\mu_r\ \sigma}}$$

Transmission Line (cont.)





Transmission Line (cont.)



$$V(z) - V(z + \Delta z) = I(z)(R\Delta z + jwL\Delta z)$$
$$I(z) - I(z + \Delta z) = V(z + \Delta z)(G\Delta z + jwC\Delta z)$$

let $\Delta z \rightarrow 0$:

$$\frac{dV(z)}{dz} = -(R + jwL)I(z)$$
$$\frac{dI(z)}{dz} = -(G + jwC)V(z)$$

"Telegrapher'sE quations"

TEM Transmission Line (cont.)

Wave Propagation on a Transmission Line

$$\frac{d^2V(z)}{dz^2} = -(R + j\omega L)\frac{dI(z)}{dz} = (R + j\omega L)(G + j\omega C)V(z)$$

$$\frac{d^2I(z)}{dz^2} = -(G + j\omega C)\frac{dV(z)}{dz} = (R + j\omega L)(G + j\omega C)\, I(z)$$

$$\frac{d^2V(z)}{dz^2} - \gamma^2 V(z) = 0$$

$$\frac{d^2I(z)}{dz^2} - \gamma^2 I(z) = 0$$

$$\gamma = [(R + j\omega L)(G + j\omega C)]^{1/2} = \alpha + j\beta$$

γ = propagation constant
α = attenuation constant
β = phase constant

TEM Transmission Line (cont.)

Wave Propagation on a Transmission Line

$$\frac{d^2V(z)}{dz^2} - \gamma^2 V(z) = 0$$

$$\frac{d^2I(z)}{dz^2} - \gamma^2 I(z) = 0$$

Solution:

$$V(z) = Ae^{-\gamma z} + Be^{+\gamma z}$$

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$

$$= |V_0^+| e^{j\phi^+} e^{-\alpha z} e^{-j\beta z} + |V_0^-| e^{j\phi^-} e^{+\alpha z} e^{+j\beta z}$$

$$= |V_0^+| e^{-\alpha z} \cos(\omega t - \beta z + \phi^+) + |V_0^-| e^{+\alpha z} \cos(\omega t + \beta z + \phi^-)$$

Reflected wave in -ve z direction

$$I(z) = Ce^{-\gamma z} + De^{+\gamma z}$$

Incident wave in +ve z direction

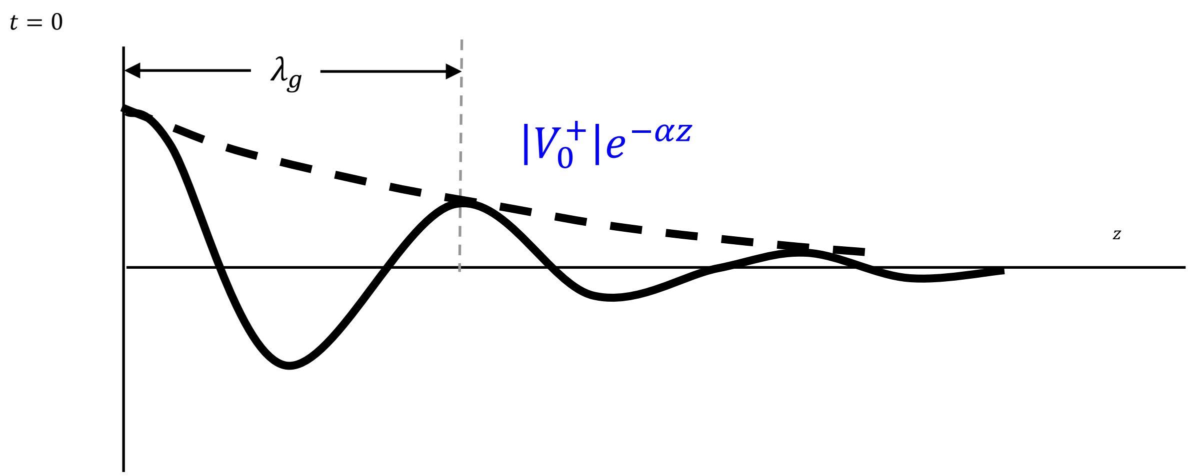
TEM Transmission Line (cont.)

Forward travelling wave (a wave traveling in the positive z direction):

$$V^+(z) = V_0^+ e^{-\gamma z} = V_0^+ e^{-\alpha z} e^{-j\beta z}$$

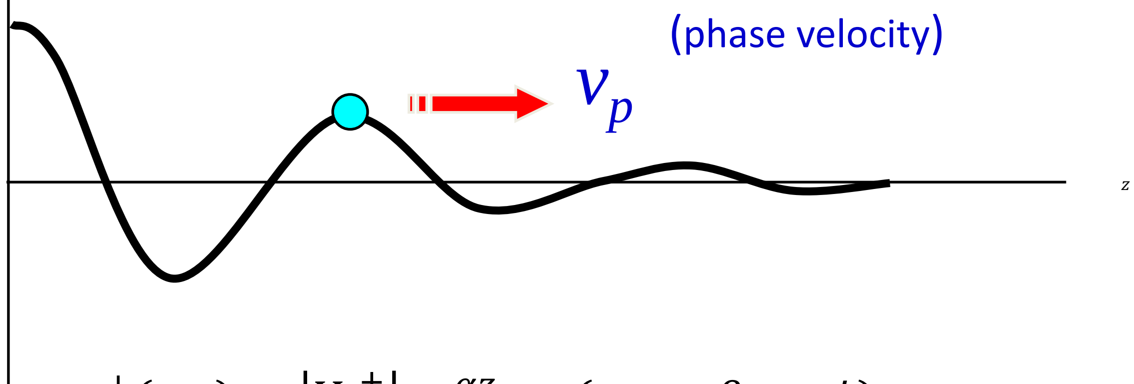
$$\begin{aligned} v^+(z,t) &= \mathrm{Re}\left\{\left(V_0^+ e^{-\alpha z} e^{-j\beta z}\right) e^{j\omega t}\right\} \\ &= \mathrm{Re}\left\{\left(|V_0^+| e^{j\phi} e^{-\alpha z} e^{-j\beta z}\right) e^{j\omega t}\right\} \\ &= |V_0^+| e^{-\alpha z} \cos(\omega t - \beta z + \phi) \end{aligned}$$

$$\beta = \frac{2\pi}{\lambda_g}$$



Phase Velocity

Track the velocity of a fixed point on the wave (a point of constant phase), e.g., the crest.



$$v^{+}(z,t) = \left|V_0^{\;+}\right| e^{-\alpha z} \cos(\omega t - \beta z + \phi)$$

Set $\quad \omega t - \beta z = \text{constant}$

$$\omega t_1 - \beta z_1 = \omega t_2 - \beta z_2$$

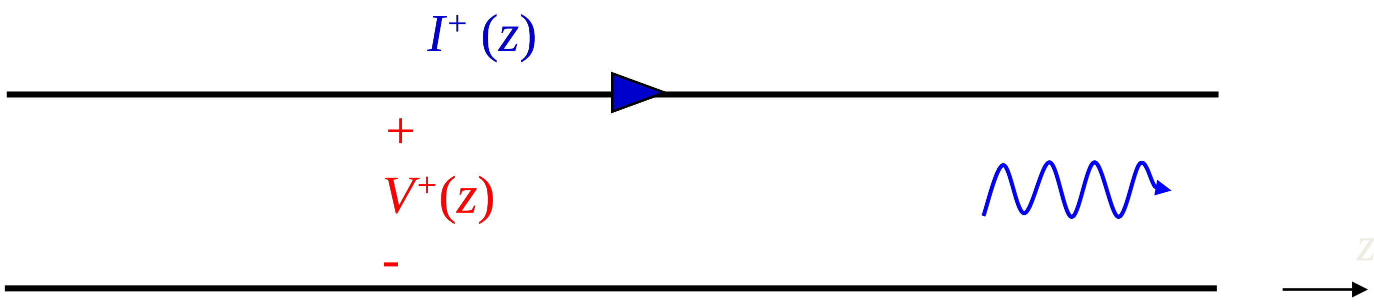
$$\frac{dz}{dt} = \frac{\omega}{\beta}$$

$$v_p = \frac{\omega}{\beta}$$

In expanded form:

$$v_p = \frac{\omega}{\mathrm{Im}\{[(R + j\omega L)(G + j\omega C)]^{1/2}\}}$$

Characteristic Impedance Z_0



A wave is traveling in the positive *z* direction.

$$Z_0 \equiv \frac{V^+(z)}{I^+(z)}$$

$$V^+(z) = V_0^+ e^{-\gamma z}$$
$$I^+(z) = I_0^+ e^{-\gamma z}$$

so

$$Z_0 = \frac{V_0^+}{I_0^+}$$

Characteristic Impedance Z_0 (cont.)

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$

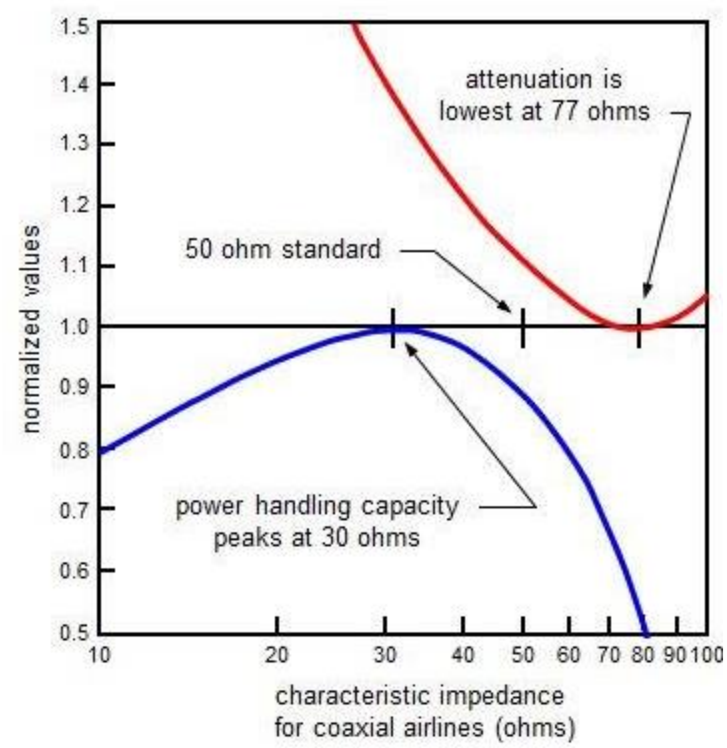
Use Telegrapher's Equation: $$\frac{dV(z)}{dz} = -(R + jwL)I(z)$$

Hence $$-\gamma {V_0}^+ e^{-\gamma z} = -(R + jwL){I_0}^+ e^{-\gamma z}$$

$$Z_o = \frac{V^+(z)}{I^+(z)} = \frac{(R + j\omega L)}{\sqrt{(R + j\omega L)(G + j\omega C)}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

Characteristic impedance of transmission line:

- a fundamental parameter of transmission line is its characteristic impedance z_0 and propagation constant γ.
- z_0 describe the relation between the current and voltage travelling waves
- z_0 is function of dimensions of T.L. and dielectric constant
- for most RF systems z_0 is either 50 or 75 ohms.(Notice z_0 is real for low losses)



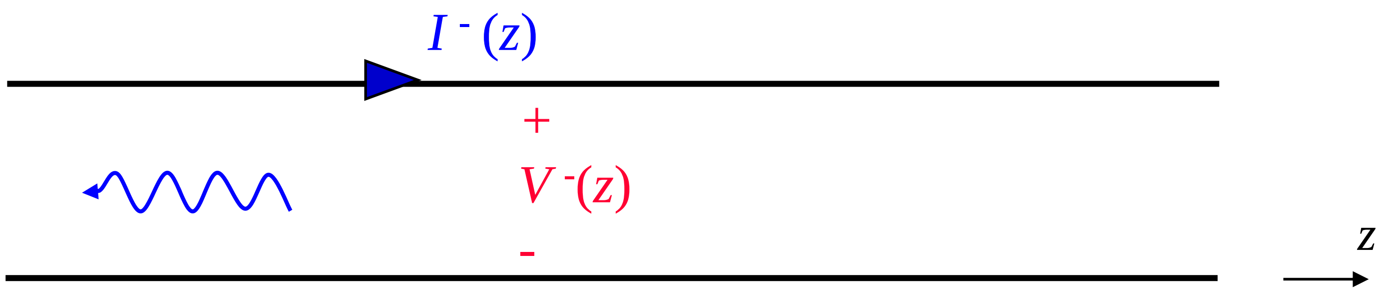
For **low power** (cable TV for example) coaxial lines are optimized for **low loss z_0 =75** ohms, for radar applications **high power** is encountered coaxial line is designed to compromise Between **max power handling and minim um losses** ,it designed to have **z_0 =50 ohm**

appendix J for standard coaxial cable, most designed for z_0 between 50-75

APPENDIX J STANDARD COAXIAL CABLE DATA

RG/U type	Impedance (Ω)	Inner cond. diam. (In.)	Dielectric material	D di
RG-8A/U	52	0.0855	P	
RG-9B/U	50	0.0855	P	
RG-55B/U	54	0.0320	P	
RG-58B/U	54	0.0320	P	
RG-59B/U	75	0.0230	P	
RG-141A/U	50	0.0390	T	
RG-142A/U	50	0.0390	T	
RG-174/U	50	0.0189	P	
RG-178B/U	50	0.0120	T	
RG-179B/U	75	0.0120	T	
RG-180B/U	95	0.0120	T	
RG-187/U	75	0.0120	T	
RG-188/U	50	0.0201	T	
RG-195/U	95	0.0120	T	
RG-213/U	50	0.0888	P	
RG-214/U	50	0.0888	P	
RG-223/U	50	0.0350	P	
RG-316/U	50	0.0201	T	
RG-401/U	50	0.0645	T	
RG-402/U	50	0.0360	T	
RG-405/U	50	0.0201	T	

Backward-Traveling Wave



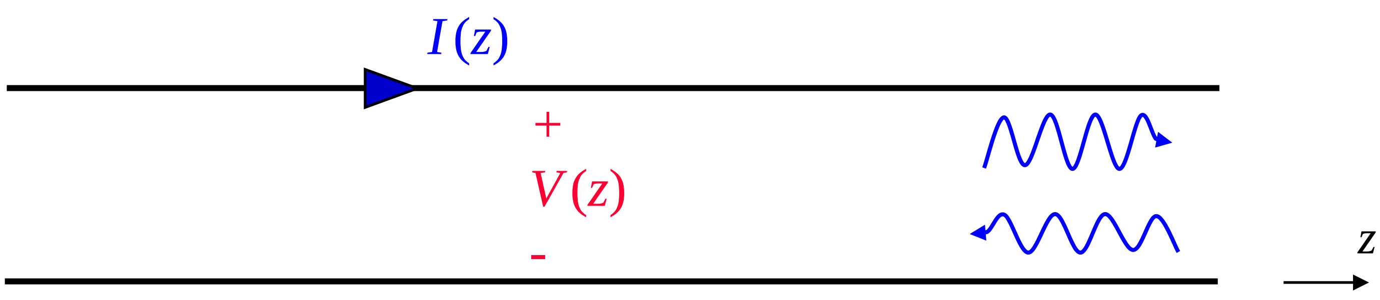
A wave is traveling in the negative z direction.

$$\frac{V^{-}(z)}{-I^{-}(z)} = Z_0$$

so

$$\frac{V^{-}(z)}{I^{-}(z)} = -Z_0$$

General Case



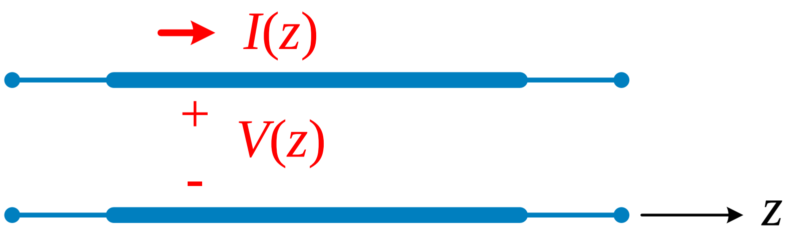
Most general case:

A general superposition of forward and backward traveling waves:

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$
$$I(z) = \frac{1}{Z_0}\left[V_0^+ e^{-\gamma z} - V_0^- e^{+\gamma z}\right]$$

Note: The reference directions for voltage and current are the same for forward and backward waves.

Summary of Basic TL formulas



$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{+\gamma z}$$

$$\gamma = \alpha + j\beta = [(R + j\omega L)(G + j\omega C)]^{\frac{1}{2}}$$

$$Z_0 = \left(\frac{R + j\omega L}{G + j\omega C}\right)^{\frac{1}{2}}$$

guided wavelength $\equiv \lambda_g$

$$\lambda_g = \frac{2\pi}{\beta} \ [\mathrm{m}]$$

phase velocity $\equiv v_p$

$$v_p = \frac{\omega}{\beta} \ [\mathrm{m/s}]$$

Lossless Case

$$R = 0, G = 0$$

$$\gamma = \alpha + j\beta = [(R + j\omega L)(G + j\omega C)]^{1/2}$$
$$= j\omega\sqrt{LC}$$

so

$$\alpha = 0$$
$$\beta = \omega\sqrt{LC}$$

$$v_p = \frac{\omega}{\beta}$$

$$Z_0 = \left(\frac{R + j\omega L}{G + j\omega C}\right)^{1/2}$$

$$Z_0 = \sqrt{\frac{L}{C}}$$

(real and indep. of freq.)

$$v_p = \frac{1}{\sqrt{LC}}$$

(indep. of freq.)

Lossless Case (cont.)

$$v_p = \frac{1}{\sqrt{LC}}$$

In the medium between the two conductors is homogeneous (uniform) and is characterized by (ε, μ), then we have that

$$LC = \mu\varepsilon$$

The speed of light in a dielectric medium is $$c_d = \frac{1}{\sqrt{\mu\varepsilon}}$$

Hence, we have that $$v_p = c_d$$

The phase velocity does not depend on the frequency, and it is always the speed of light (in the material).